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**THESIS TITLE**

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**Authorship Statement**

**This dissertation is based on the results of research carried out by myself, is my own composition, and has not been previously presented for any other certified or uncertified qualification**

**This research was carried out under the supervision of <Supervisor Name>**

**DATE – SIGNATURE**

**Copyright Statement**

**In submitting this dissertation to the…**

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**ACKNOWLEDGEMENTS**

**I would like to express my sincere gratitude to my mentor…**

**ABTSTRACT**

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**List Of Abbreviations**

**P** Polynomial Time

**NP** Non-deterministic Polynomial Time

**TSP**  Travelling Salesman Problem

**Literature Review**

**P vs NP**

The P vs NP problem is one of the seven “Millenium Problems”, which are a set of famous unsolved problems in mathematics, founded by Landon T. Clay in 1998 [1]. In principle, the P vs NP problem asks whether every problem whose solution can be quickly verified can also be quickly solved.

**2.1 Polynomial Time**

P(Polynomial Time) refers to the class of decision problems that a deterministic algorithm can solve in polynomial time – meaning that the time required to solve the problem grows at most as some fixed power of the input size [2].

A graph of different colored arrows

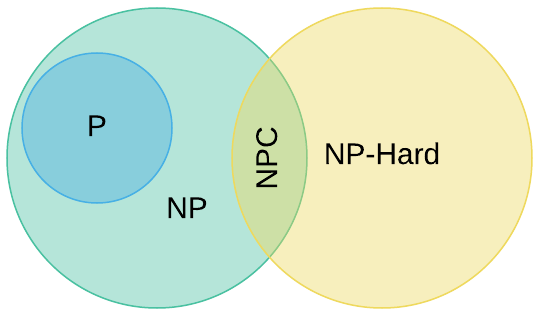
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**Figure 1**: Time Complexity [3]

Figure 1 illustrates common algorithmic time complexities. While quadratic time, O(n²), is relatively inefficient compared to linear or logarithmic time, it still falls within the class of polynomial-time algorithms. In contrast, exponential time, O(2ⁿ), and factorial time, O(n!), grow much faster and lie outside the class P

**2.2 Non-deterministic Polynomial Time**

NP(Nondeterministic Polynomial Time) refers to a problem for which a given solution can be verified in polynomial time by a deterministic algorithm – even if finding that solution could prove difficult [3].



**Figure 2**: Complexity Classes [4]

Figure 2 shows the complexity classes working together, it clearly depicts P ⊆ NP always. The table below describes the key relationships between the class and its function (NP-Complete and NP-Hard are followed through in sections 2.3 and 2.4 respectively):

| **Class** | **Description** |
| --- | --- |
| **P** | Problems solvable quickly (polynomial time) by deterministic algorithms. |
| **NP** | Problems whose solutions can be verified quickly, includes all of P. |
| **NP-Complete** | The hardest problems within NP. Efficiently solving one implies efficient solutions for all NP problems. |
| **NP-Hard** | Problems at least as hard as NP-complete, but not necessarily in NP-or even decidable. |

**Table 1**: Complexity Class Functions

**2.3 NP-Complete**

NP-complete problems are a subset of NP with two defining properties:

* **In NP** - Their solutions can be verified in polynomial time by a deterministic algorithm.
* **NP-hardness** - they are at least as hard as any problem in NP, meaning every NP problem can be transformed (reduced) to them in polynomial time [5].

Such example of an NP-Complete problem is the Traveling Salesman Problem (TSP), given a list of cities and the distances between them, the task is to determine the shortest possible route that visits each city exactly once and returns to the starting point. Verifying a proposed route’s total distance is fast (polynomial time), but finding the optimal route may require checking exponentially many possibilities - making TSP a classic NP-complete problem [5]

**2.4 NP-Hard**

NP-hard problems are those to which *every* problem in NP can be reduced in polynomial time, making them at least as challenging as NP-complete problems. Importantly, NP-hard problems are not required to be in NP—they may lack efficient verifiers or even be undecidable. In contrast, NP-complete problems must be both in NP (verifiable in polynomial time) and NP-hard, placing them among the most difficult problems within NP [6].

Such example of an NP-Hard problem is the Halting Problem, which asks whether a given computer program will eventually stop running (halt) or continue executing forever, for a specified input. Alan Turing proved in 1936 that there is no general algorithm capable of solving this problem for all possible program–input pairs, making it undecidable [7].

**1.5 P vs NP Conclusion**

Hence, P = NP is equivalent to stating “Is every problem whose solution is easy to check, also easy to solve?”. This question is one of the most important open problems in theoretical computer science, with far-reaching implications. If P = NP were proven true, a wide range of problems currently thought to be intractable could be solved efficiently, revolutionizing fields such as cryptography, optimization, logistics, artificial intelligence and bioinformatics amongst others. Conversely, if P != NP, it would confirm that certain problems are incapable of efficient solutions.

**Expressions Explainability in Appendix?**

P ⊆ NP – Every problem in P is automatically in NP.

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